Centroids!

Calculus 12, Veritas Prep.

The **centroid** (or center) of the region between the functions $f(x)$ & $g(x)$ and the lines $x = a$ & $x = b$ is given by the following coordinates:

$$x_c = \frac{\int_a^b x[\, f(x) - g(x)\,]\, dx}{\int_a^b f(x) - g(x)\, dx}$$

$$y_c = \frac{\frac{1}{2}\int_a^b (f(x))^2 - (g(x))^2\, dx}{\int_a^b f(x) - g(x)\, dx}$$

or, put differently:

$$\left(\frac{\int_a^b x[\, f(x) - g(x)\,]\, dx}{\int_a^b f(x) - g(x)\, dx}, \frac{\frac{1}{2}\int_a^b (f(x))^2 - (g(x))^2\, dx}{\int_a^b f(x) - g(x)\, dx} \right)$$

Sketch the regions enclosed by the following curves and then find their centroids. (Mark the centroids on your sketches.)

1. $y = 2 - x$, $y = 0$, and $x = 0$.
2. $y = 2 - x^2$, $y = 0$
3. $y = \frac{1}{3}x^2$, $y = 0$, $x = 4$
4. $y = x^3$, $y = 0$, $x = 1$
5. $y = \frac{1}{2}(x^2 - 10)$, $y = 0$, $x = -2$, $x = 2$
6. $y = 2x - 4$, $y = 2\sqrt{x}$, $x = 1$
7. $y = x^2$, $y = x + 3$
8. $x = y^2$, $x = 2$
9. $x = y^2 - 3y$, $x = -y$
10. $y = x^2$, $y = 4$.
11. $y = x^k$, $y = 0$, $x = 1$ (where k is an integer greater than zero)
12. $y = x^m$, $y = \sqrt[n]{x}$ (again, where m and n are integers greater than zero. What happens if $m = n$?)