A Weekend In Arizona

Calculus 12, Veritas Prep.

This is **due on Monday, December 6th**. (I'll be collecting and grading it.) Do not wait until Sunday night to start it—you won't finish. Please write up your answers on a separate sheet of paper (clearly headed with your name and the date, and with each problem clearly labelled). Please take the time to write it up *nicely*—don't hand in a paper full of scratch work. Use sentences, explain your methodology, and justify each of your steps in English and in math. Use as a model the essay I gave you a few weeks ago, "How to Write Math in Paragraph Style," by Tim Hsu[1]. I encourage you to collaborate with your classmates, but the answers you write up should be your own. Feel free to use a calculator.

You are trying to drive the 15 miles to Veritas from your house in Tempe, but the traffic on the 202 is awful—it is total stop-and-go—and so your speed fluctuates sinusoidally between 24 mph and 0 mph. Put a bit more formally, your speed (or velocity) as a function of time is $v(t) = 12\sin(20\pi \cdot t) + 12$ mph (where t is in hours). **(1)** Find the total amount of distance you cover in 10 minutes. **(2)** Find the total amount of distance you cover in t minutes. **(3)** How long does it take you to get to school? (You might have to estimate using a calculator.)

Later, over the weekend, you drive to Prescott in order to show off your proper pronunciation of the name to the townies. But on one of the flat stretches of I-17 near Agua Fria, you run out of gas, and your car slowly rolls to a stop. At the moment you run out of gas, you are driving 90 mph, and without gas, the speed of your car decreases autoproportionally[2], such that after one minute *sin petrol* you are driving only 45 mph. **(4)** Find an equation for your speed (after you run out of gas) as a function of time. (You can do this without calculus, but I want you to do it with calculus.) **(5)** How fast will you be going after 2 minutes? 3 minutes? what about after 6.4 minutes? **(6)** When will you be going only 1 mph? (Note that we're ignoring friction in this problem, and assuming the road is perfectly flat.) **(7)** Find how much distance you cover in 10 minutes of coasting.

Once you get to Prescott, you go to a local coffee shop and down, in quick succession, five shots of espresso. One shot (= 30mL) of espresso contains approximately 100mg of caffeine (molecular formula $C_8H_{10}N_4O_2$). Imagine that the concentration of caffeine in the body follows the following differential equation[3]:

$$\frac{dy}{dt} = y\left(\frac{1}{t} - k\right)$$

where y represents the concentration of the caffeine in the blood, and k is some positive constant, which for caffeine equals about $.01\,L/mg/min$. **(8)** Come up with an equation for the concentration of caffeine in your bloodstream as a function of time. (Be sure to solve for the constant—which is hard. It might help to contemplate the fact that the maximum concentration of caffeine will be $500mg/5L$, where $5L$ is roughly the amount of blood in an adult human.) **(9)** How many caffeine molecules do you ingest? **(10)** Based on this model, when will your veins be fully expunged of caffeine? (This is hard, too—in fact, it's mathematically impossible. But that doesn't mean it's impossible—you can use your calculator in a creative way to estimate it.)

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[1] also online at `http://www.math.sjsu.edu/~hsu/`

[2] I have no idea if this is a word, but I wanted to use it. What I mean is just that your speed decreases proportionately to itself—i.e., that $\frac{dv}{dt} = -kv$ for some positive k.

[3] I'll admit that I basically made this math up all by myself, but only after an evening of doing heavy background research—I needed math that would challenge you in a very specific way, but also wanted it to be grounded in reality. It's really interesting stuff—you should Google (or Google Scholar) "caffeine," "drug clearance," and "pharmacokinetics." I learned a whole lot.