

The Meno (somewhat adapted and abbreviated)

Socrates: By the gods, Meno, be generous, and tell me what you say that a limit is; for I shall be truly delighted to find that I have been mistaken, and that you and Leibniz do really have this knowledge; although I have been just saying that I have never found anybody who had.

Meno: There will be no difficulty, Socrates, in answering your question. Let us take first the limit of a rational function—if a rational function has a hole, one must eliminate the hole, and then plug in numbers. The limit of a polynomial, if you wish to know about that, may be easily described: all you do is plug the number into the function. Sometimes the limit is the true value of the function, but sometimes it differs. There are also limits from the left side and limits from the right side. Every function, every point, large or small, rational or integral, real or complex, has a different limit: there are limits numberless, and no lack of definitions of them; for a limit is relative to the actions and axes of each particular function at each particular point.

Socrates: How fortunate I am, Meno! When I ask you for one limit, you present me with a swarm of them, which are in your keeping. Suppose that I carry on the figure of the swarm, and ask of you, What is the nature of the bee? and you answer that there are many kinds of bees, and I reply: But do bees differ as bees, because there are many and different kinds of them; or are they not rather to be distinguished by some other quality, as for example beauty, size, or shape? How would you answer me?

Meno: I should answer that bees do not differ from one another, as bees.

Socrates: And if I went on to say: That is what I desire to know, Meno; tell me what is the quality in which they do not differ, but are all alike; would you be able to answer?

Meno: I should.

Socrates: And so of the limits, however many and different they may be, they have all a common nature which makes them limits; and on this he who would answer the question, “What is a limit?” would do well to have his eye fixed. Do you understand?

Meno: I am beginning to understand; but I do not as yet take hold of the question as I could wish.