

Write Your Own Adventure

Calculus 11, Veritas Prep.

Imagine this scenario: you're a student at an academically-demanding school in the northern Sonoran desert, and you've been doing word problems with derivatives for the last month or two, except the word problems are really rather bland, and you're good enough at the fundamental mathematics that you can do all of them without much trouble. They're so boring and so tedious that whenever you sit down to do them, you always end up zoning out. You sit at your desk, idly tapping your pencil and staring at the wall, all while wishing: *if only this stuff could be harder and more interesting!*

This is not a hypothetical situation! This is, in fact, your final project for this class: I want you to write and solve a word problem. It might sound easy, but in many ways is quite a bit harder than just solving a word problem. Often in the 12th grade class I have spent several days designing a single word problem (and I can do most of the symbol-manipulation in my head). The math in your problem needs to be all correct. But that's just the basics. When you write a Humane Letters essay, the English needs to be sensible and syntactical. You have plenty of time at home to check your algebra and calculus. You shouldn't get any of it wrong. What I'm really looking for is for you to do two things: one, create a reality that induces a challenging, non-trivial math problem, and two, solve it. Put differently, **I want you to write and solve a very creative word problem that requires difficult mathematics.**

As examples, consider the following four word problems:

Socrates walks into a coffee shop and the barista asks him what $9 + 1$ is.

This is terrible. The math is entirely *deus ex machina*—it is not remotely related to Socrates or a coffee shop. Plus, it's utterly simple—no one taking calculus should have the slightest bit of difficulty adding two one-digit numbers.

Socrates and Plato walk into a coffee shop. Socrates orders nine shots of espresso and Plato orders one shot of espresso. How many shots of espresso do they order in total?

This is marginally better. The math is actually integrated into the narrative, but still, $9 + 1$? The math needs to be challenging.

Socrates walks into a coffee shop. The barista asks him to find the volume of the solid generated by revolving the functions $y = \sqrt[3]{x+4} - 2x$ and $y = 3\sin(2x^4)$ about the line $y = x$.

OK, so here we have math that is actually substantive. This is a difficult problem. For one thing, the functions are complicated, which will probably make the algebra unpleasant. But more importantly, how do we revolve a shape around a slanted line? We know how to revolve shapes around axes or lines parallel to the axes—but how do we deal with a slanted line?¹ Anyway, the math in this problem is good, but it's still totally unrelated to Socrates's coffee break—and as a result, fails as a word problem.

Socrates walks into a coffee shop... but then the barista tells him that the Sophists are hot on his tail and he needs to flee Athens. So Socrates runs outside and jumps into his Toyota Corolla—maybe not the best getaway car, but sensible and affordable. After all, who is Socrates trying to impress? He'd rather donate the profits from his academy to charity. Anyway, as he's racing down the street, sending money-changers and street urchins flying, he looks in his rear-view mirror and sees three F-22 fighters from the elite Sophist Air Superiority Arm in pursuit. One of them fires a Hellfire missile. At the moment the missile is fired, Socrates is driving 15 mph, and his car can accelerate at 14 meters per second per second (m/s^2) up to a top speed of 75 mph. The Hellfire, on the other hand, can accelerate at 40 m/s^2 up to a top speed of Mach 1.3. Question: when does the vial of hemlock Socrates has stored in his wallet become irrelevant?

¹Go online and look up "rotation matrix" if you're curious—it will look really scary, because it pretends to be a matrix, but it's really nothing more than a messy formula. You can understand it without thinking of it as a matrix, and if you had enough time on your hands, you could probably come up with it by yourself.

This is good. The math is difficult, and is clearly related to the very vividly written story. (Whether the story is an accurate interpretation of Socrates's teachings or death remains an open question.)

I hope this gives you an idea of the kind of problem I'm looking for. There are plenty more examples in the problems I've given you this year and the problems in your book. I want you to challenge yourself and produce something you're proud of. This is a way of pulling together everything you've learned this year into a unified whole.

Your word problem should involve calculus, which means that it should probably be a related rates problem and/or an optimization problem. One especially cool thing you might try, if you're ambitious, would be to combine the two—write some sort of a problem that involves finding a maximum/minimum rate.

When you turn it in, be sure it is written up neatly (typed is best), and that your solution to the problem is clear, carefully-explained, and complete (so that even your younger siblings can understand it)². I'm giving you this assignment quite a while before it's due, so you are welcome to turn it in early. (Do not wait until the night before to start—you will regret it.)

Due: May 2nd

²As a reference/guide, you might want to read the article "How to Write Math in Paragraph Style," by Tim Hsu, online at <http://www.math.sjsu.edu/~hsu/>