



March 6, 2020

FIRSTNAME LASTNAME  
Math 3, Block XXXX  
The Nueva School

Dear FIRSTNAME,

I'm writing to tell you about a letter you're going to write your imaginary penpal at the Vieja School in West Dakota. Your penpal, like you, has been studying complex numbers, but due to a tragic branch-cutting accident their class has been discontinued, right before they were about to learn about taking the roots of complex numbers. They still want to learn how to take complex roots, and you need to help them!

Write them a letter that walks them through two examples of finding (and then verifying) complex roots. Don't just give them that huge formula that we came up with at the end (the one about all  $k$   $n$ 'th roots of any complex number) and plug things in. That's not learning! That's not teaching! That's not *real understanding*! Rather, explain to them, geometrically and in polar form, how to find complex roots, in a way that helps them really *understand* what's going on. Be sure to demonstrate to your pen pal that the solutions you come up with actually are the right roots: imagine your friend is skeptical, and so show algebraically and geometrically that the roots exponentiate to produce the original number.

Your first example should be of finding all the cube roots of a number that's on the unit circle (keeping things nice and simple). Your second example should be finding all of some higher-than-cube roots of a number that's *not* on the unit circle (e.g., all the quartic roots, or all the vigintiseptic roots, etc.; choose one set of roots) and that's not 1.

I've got a couple more tips. Although your penpal is pretty good at polar coordinates and everything involving complex numbers up until taking roots, unlike you, they're not all that quick on the take. So you'll need to do *a lot of explaining*. Make sure you explain everything as thoroughly and as clearly as you can, using mathematical symbols, pictures, eloquent English prose, and so forth. Also, if you include big equations or chunks of math in your letter, put them on their own line. It can get hard to read mathematics when it's in-line with English prose. (You can look at my notes for examples of how to do that.)

I'll collect your letter on **Monday** or **Tuesday, March 16th or 17th**, during your block, and then mail it to your penpal.

Remember: just because your penpal is imaginary doesn't mean they're not real!

Yours in abstraction,