Let's Draw Powers of i And Stuff!!!

Math 3

Some problems to do, right now, on a separate sheet of paper:

4-1. On the complex axes, plot the following points:

- i^0
- $\sqrt{i}$ a/k/a $i^{1/2}$ (We figured out in class what this is! Remember, just like with all other square roots, there are *two* of them.)
- i
- i^2
- i^3
- i^4

4-2. As we're plotting i^k, where k is starting at 0, going up through 1/2, to 2, 3, 4, etc., what's happening?

4-3. Any guesses as to where $\sqrt[3]{i}$ a/k/a $i^{1/3}$ might go on these axes? (I guess you already figured out, on a different worksheet and in your lovely writeups, what $\sqrt[3]{i}$ is. Plot it anyway. Any patterns you see?)

4-4. Plot $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$

4-5. Plot $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$

4-6. Plot $\dfrac{\sqrt{3}+1}{2\sqrt{2}} + \dfrac{\sqrt{3}-1}{2\sqrt{2}} \cdot i$

4-7. What power do you think you have to raise i to to get $-\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i$? (You can probably find one power pretty easily. What's another power? A third?)

4-8. What power do you think you have to raise i to to get $-\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i$? (Again, can you find multiple powers that work?)

4-9. What if you wanted to make a power of i that gives you $-\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}i$? What power would that need to be? In other words, what's the solution to:

$$i^{?????} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

(Once you find one solution—is that the only one? Are there any others? How many are there?)

4-10. Consider these two complex numbers:

$$z_1 = \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot i$$

$$z_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

You should already have drawn them on your complex plane, but if your current complex plane is getting messy, draw them on a new one. Then, multiply them together. Plot the result. What do you notice?

(If you weren't in my Math 3 section last semester, ask someone who was if there's anything interesting about the number(s) $\frac{\sqrt{3}\pm 1}{2\sqrt{2}}$.)

4-11. Now consider these two complex numbers:

$$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$z_2 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

Again, plot them on a new set of complex axes. Then multiply them together, and plot the result. Any observations?

4-12. Here's another pair of complex numbers.

$$z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i \qquad z_2 = \frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot i$$

Divide them (z_1/z_2). What do you get? Is it familiar?

4-13. Suppose I ask you to multiply together the following complex numbers:

$$\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) \cdot \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \cdot (-i) \cdot \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) \cdot \left(\frac{\sqrt{3}+1}{2\sqrt{2}} + \frac{\sqrt{3}-1}{2\sqrt{2}}i\right)$$

(a) Do you want to multiply all of these out by hand?
(b) Do you have any better ideas?
(c) Are you lazy? Can you figure out a faster way?
(d) Don't just "suppose" that I asked you to multiply those numbers together—actually multiply them together. What do you get?

4-14. Suppose you have a fractional power of i, like $i^{3.2}$ or $i^{12/11}$, where the exponent isn't an integer. Is the resulting number purely imaginary, purely real, or non-real non-imaginary complex? Why or why not?

4-15. Think back to the problem we did last week where we found all six sixth roots of 64.

(a) One of the roots of 64 we found was $1 + i\sqrt{3}$. Can you multiply that by itself six times, and plot the result of each successive multiplication? In other words, plot:

$$\left(1+i\sqrt{3}\right), \left(1+i\sqrt{3}\right)^2, \left(1+i\sqrt{3}\right)^3, \left(1+i\sqrt{3}\right)^4, \left(1+i\sqrt{3}\right)^5, \left(1+i\sqrt{3}\right)^6$$

You may as well plot $\left(1+i\sqrt{3}\right)^0$, too. What happens? Describe.

(b) Is this the same (conceptually, more or less) as all the previous problems on this worksheet? If not, how is it different?
(c) All of this multiplication is so unpleasant. Is there a pattern that would let us plot all six of those points above without having to do so much tedious binomial expansion???
(d) What happens if we repeat this same process with each of the other five sixth roots of 64? Does the graph look the same, or different? How?