

Some Themes in 11th Grade Math

Calculus 11, Veritas Prep.

Abstraction and generality. We know how to find the sine/cosine/tangent of angles between 0 and 90 degrees—but can we find the sine/cosine/tangent of negative angles? what about angles greater than 90°? We know the Pythagorean theorem for right triangles—can we extend it to *any* triangle? We know how to deal with functions of the form ax^n where n is a natural number (i.e., polynomials)—what if n is any integer (and thus ax^n a rational function?) what if n is a rational number (and this ax^n a root)? We know how to find the slope of a straight line—can we find the slope of a curvy line? If we have a function, we know how to take its first/second/third/etc. derivative—but what would it mean to take its -1 st derivative? its -2 nd derivative? what about its three-and-a-halfth derivative?

Formalism. We have a rough idea of how to sketch the slope of a curvy line—but how do we find the equation of the slope? We have some idea of what we mean by “limit”—but how can we state this idea in a logical, mathematical way? We suspect that the derivative of a function x^n is nx^{n-1} ; can we prove this? We know how to find the area of circles, rectangles, and triangles—but what do these “areas” have in common? What is the fundamental idea of “area” beneath each of these particular instances? Can we express it mathematically? We have intuitive concepts in our minds (math, morals, truth) and we want to formalize them (into equations, laws, syllogisms). When the intuitive and the formal versions are in conflict, which one is right?

Indirect knowledge. It’s hard to factor polynomials, but it’s easy to check that our factorization works—just multiply it out. Does it make sense to define things “non-constructively”? Does it make sense to define a limit (using ϵ s and δ s) in a way that tells us whether a given number is a limit—but not what a limit actually *is*? Is there a formula for antidifferentiation, or is an antiderivative of a function just something that, when you differentiate it, gives you the function?