Calculus 11/12

Veritas Preparatory Academy
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I intend this to be a rough overview of how I've taught this course in the past and how I expect to teach it this year. Consider it descriptive, rather than prescriptive. I may teach somewhat differently. There may be times when I will ask you to do things, the reason for which may not be immediately clear. I ask that you trust that I have your best mathematical and educational interests at heart, and I ask that you put forth your best effort.

Calculus. Indeed, this is a course on calculus! However, the syllabus is the place for administrative details, and I do not wish to dirty calculus by placing it in such a (comparatively) vulgar context. Talking about calculus is what we will do every day of the year. All I will say is that we will study calculus, it will be very exciting, and by the end of the year you will be extremely good at it.

Texts. There is, for the most part, no published text. We will be using my notes and problem sets. Please read the notes—I put a lot of effort into making them *readable*, unlike most math texts. If you have questions or are confused, they should be the first place you turn. I also encourage you to use other resources beyond my notes to help you learn calculus. The library and the internet are full of excellent books, tutorials, and videos that can give an additional perspective, and I am happy to offer recommendations. (For the logic and algebra units, we'll be using the first two chapters of Velleman's *How to Prove It*, as well as the bulk of Maxfield and Maxfield's *Abstract Algebra and Solution by Radicals*.)

Homework. I will try to explain calculus to you to the best of my abilities, but there is no way I will be able, through sheer force of lecture alone, to make you *see* and *feel* the patterns and beauty of calculus. This is why I will assign lots and lots of problems for you to do. Some will be rote; others will require serious contemplation and genuine creativity. The more and more of these problems you solve, the more and more you will understand the mosaic-web of this beautiful mathematical structure. I can give you guidance, and I can help push you up some of the steep parts, but a great deal of the work you are going to have to do yourself. (Analogously, I might be able to give a brilliant and insightful lecture on some book, but to really understand it, you'd need to pour over the book again and again and again, by yourself and in conversation with others, nonlinearly, haltingly, and possibly with great frustration.)

Tutoring. But that does not mean that you should be perpetually frustrated. If you have questions that are not answered in class, in your outside discussions with classmates, or in the textbook, please come talk to me. I am always happy to tutor before and after school (though I might not always be free).

Participation. I need you to help me teach this class. I expect you to ask relevant questions, volunteer answers, share insights and observations during class discussion, and participate in class activities. I might call on you even if you are not raising your hand. I might ask you to present problems in front of the class. I intend this class to be an active conversation about calculus.

Calculators. You will not need them.

Grading. Please do not ask me how you can get a better grade. But please *do* ask questions like, "How can I understand calculus better? How can I be a better student in your class?" What matters is the amount of effort you put in, and the degree to which you challenge yourself. If you ask me about your letter grade, I will be unhappy. And you do not want the person who grades you to be unhappy with you. I will say but this: everything you do in this class I take into consideration when writing your narrative evaluation. That includes (in no particular order) your tests, homework, papers, participation, presentations, attitude, curiosity, creativity, effort—everything.

Syllabus

11th grade

Summer: Exponentiation. Algebra (solving polynomial equations, proofs of binomial theorem, the quadratic equation and its derivation). Baby algebraic geometry (parent functions and their linear transformations, sketching polynomials, sketching rational functions). Logarithms.

Fall: Trigonometry. Right-angle and unit circle definitions of trig functions, exactly evaluating trig functions, periodic/symmetry identities of trig functions, sum and difference identities, the law of sines, the law of cosines (all with proof), equations with trig functions, word problems! Derivatives. Intuitive slope-sketching, construction of Fermat's difference quotient, basic derivative computations and proofs. Limits, informally. Limits, formally, using the ϵ-δ definition.

Winter: Basic derivative laws and proofs thereof (sum/difference/product/quotient rules, derivatives of trig functions, the chain rule). Implicit differentiation. Derivatives of logs and exponential functions. Related rates word problems.

Spring: Optimization. The idea of integration. The proof of the Fundamental Theorem of Calculus. Antiderivatives!

12th grade

Summer: Integrals and their applications—areas under curves, areas between curves, centroids, solids of revolution, possibly fluid force. Improper integrals (and technical aside for L'Hopital's rule).

Fall: Differential equations (basic, separable, first-order linear, Newtonian kinematics) and related word problems.

Winter: Infinite series. Sequences of numbers and functions and their convergence, series of numbers and functions and their convergence, Taylor series, fun with Taylor series. Introduction to mathematical logic.

Spring: Continued logic (Boolean and quantificational logic), basic set theory, incompleteness, transinfinite cardinals, abstract algebra and basic algebraic structures (groups, rings, fields).